INSTABILITIES DIAGNOSIS AND THE ROLE OF K IN MICROWAVE CIRCUITS

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ABSTRACT

Many papers, textbooks and the leading CAD packages state that a Two-Port is stable if and only if K>1 and $|\Delta_s|<1$ (or an equivalent set of conditions). The stipulation that the statement is rigorous only if no poles of the unloaded circuit lie in the right half plane seems lost on current microwave designers who rely on K and $|\Delta s|$ for determining the stability of their designs. Examples showing oscillating circuits with K>1 and $|\Delta s|<1$ are shown. The role of K as well as methods for diagnosing circuit stability are discussed.

INTRODUCTION

A dichotomy exists between the approaches of microwave engineers and control engineers to circuit design.

Microwave designers rely heavily on the so-called stability factor K in ascertaining the stability of their designs. Several variants of sets of conditions exist in the literature. All these sets of conditions, which were derived by steady state analyses, have been shown to be equivalent [1-7]. Many textbooks [8-10] and the leading software packages make the following or equivalent statement: A Two-Port is unconditionaly stable if and only if, for all frequencies ω, K>1 and alternatively $|\Delta_s|<1$ or $B_1>0$ or $1-|s_{ii}|^2 > |s_{12}s_{21}|$ for i=1,2 where:

$$K = (1-|s_{11}|^2-|s_{22}|^2+|\Delta_s|^2)/2|s_{12}||s_{21}|$$
$$B_1 = 1+|s_{11}|^2-|s_{22}|^2-|\Delta_s|^2$$

and $\Delta_s=s_{11}s_{22}-s_{12}s_{21}$ is the determinant of the S-parameters matrix. The same conditions can be stated in terms of other circuit parameters such as Z, Y, H etc. where k takes the invariant form:

$$K = \{2\mathrm{Re}(\gamma_{11})\mathrm{Re}(\gamma_{22}) - \mathrm{Re}(\gamma_{12}\gamma_{21})\} / |\gamma_{12}\gamma_{21}|$$

and absolute stability is claimed if and only if $\mathrm{Re}(\gamma_{11}) >0$, $\mathrm{Re}(\gamma_{22}) >0$ and K>1.

Control engineers, on the other hand, are faced with the task of designing circuits which are quick to respond to any perturbation. To this end they concentrate on the transient responses of their circuits and make sure that their designs do not contain any poles of $S = \sigma + j\omega$ which lie in the right half of the plane i.e with $\sigma > 0$. As is well known, the poles of any circuit give rise to transient time responses of the form $t^n e^{\sigma t}\sin\omega_0 t$ which appear at all the nodes of circuit and which die out in time, if and only if $\sigma<0$. The poles of the circuit are given by the roots of the determinant of the matrix which fully describes the circuit's behavior. Either direct computer solutions of the determinants or polar plots in the complex plane of properly defined functions which are related to the determinants (such as Nyquist's [11] plots) can be used to ascertain the location of the roots.

As it turns out, the approach undertaken by the microwave designers with its reliance on K is severly limited in many cases. The limitation is that the analysis does not hold in general, and fails in many cases, if the unloaded circuits under investigation contain poles with positive real parts, i.e. poles in the right half plane. Although the limitation of using steady state analysis in investigating the stability of Two-Ports has been recognized in Rollett's [4] paper, he does not emphasize it and mentions it only in passing. More importantly, it has been completely neglected by more recent publications, textbooks and software vendors and forgotten by microwave designers.

A proper statement of the Two-Port stability criteria involving K should be: ***An unloaded Two-port which has no poles in the RHP will remain stable when loaded externally at its input and output if and only if K>1 and $|\Delta_s|$ <1 for all ω.*** The role of using K in determining the stability of Two-Ports is seen to be quite diminished. It is limited to the investigation of loading which does not cause stable unloaded circuits to become unstable. The stability of the open circuit has to be ascertained by other means.

It is interesting to note that the design problem addressed by K and $|\Delta_s|$ is similar to the original control problem addressed by Nyquist [11]. In both cases answers are sought to the problem of determining the conditions under which individually stable circuits can be connected to create more complex stable circuits. In the control case it was the closing of the open loop to create feedback and in the Two-Port amplifier case it is the permissible terminals loading. In both cases the stability of the starting components has to be ascertained by other methods.

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Two examples will be given to illustrate the limited role of K in determining the stability of microwave circuits. The first example is of an amplifier circuit which was designed and manufactured in MMIC form and the other is a simple study model of a ring oscillator. Both circuits fullfilled all the conditions of K>1 and $|\Delta_s|$ <1 for all ω but nevertheless exhibit strong microwave oscillations.

AMPLIFIER WITH STRONG MICROWAVE OSCILLATIONS

The schematic diagram of the amplifier is shown in Figure 1. It is a wide band power amplifier composed of two reactively matched 2-stage cascades in parallel. The topology of the design is similar to that presented recently by Freitag [12] who observed that certain so-called odd mode instabilities cannot be predicted by K. His method of analysis of these oscillations, although very elegant, is not general enough since it is limited to symmetric cases. We found that strong oscillations can exist even when the circuit is highly non symmetric by applying unequal biases to the devices.

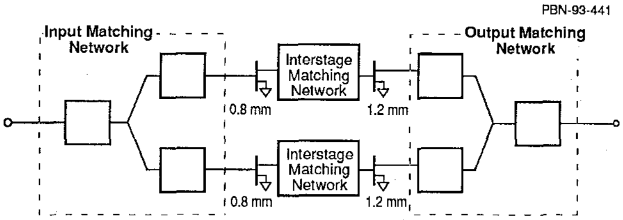


Figure 1. Topology of 2-Stage Wide Band Power Amplifier.

A spectrum analyzer photograph of the strong microwave oscillations with a fundamental at ~8 GHz is shown in Figure 2a. Figure 2b shows k and $|\Delta_s|$ vs. frequency which were calculated from our circuit design file when the devices are biased symmetrically. The figure shows that K>1 and $|\Delta_s|$<1 for all ω. Verification of the predictability of the oscillations is shown by the return ratio [13,14] calculations around each FET shown in Figure 3. Instabilities arising from a pole in the RHP are predicted if the frequency polar locus of the return ratio encircles the point -1 or alternatively, if the magnitude of the return ratio is greater than 1 while the phase passes through 180°. Any modelling inaccuracies arising from the transfer of the design to MMIC form do not affect our conclusion about the failure of K to predict the instabilities since a single design file was used for the curves of Figures 2 and 3. Non symmetric biasing gave rise to very similar results and conclusions regarding K.

The return ratio plot is very similar to Nyquist's plot and is a particular implementation of complex plane contouring of a function related to the system determinant alluded to in the introduction. A more general method for the detection of poles in the RHP which involves another function related to the system determinant is shown in the next circuit example.

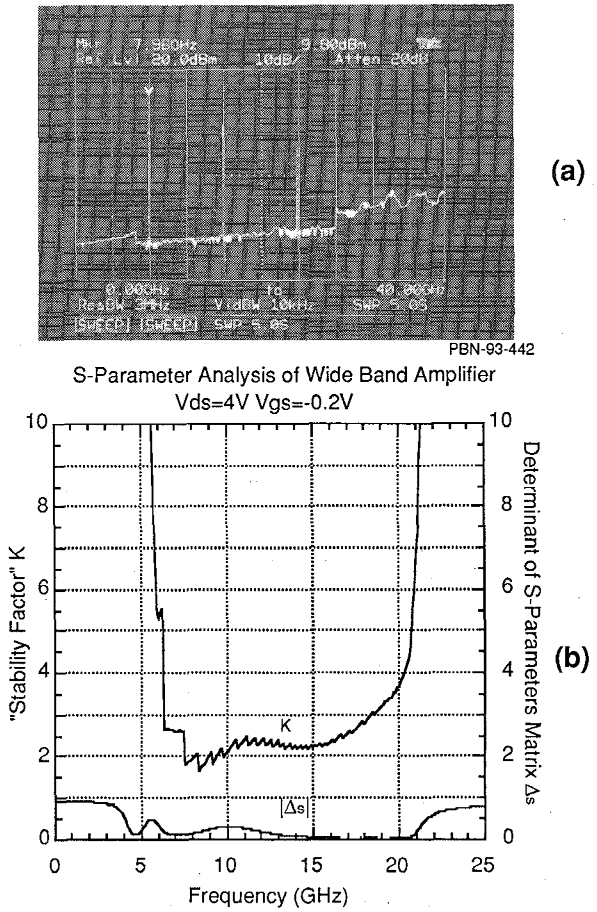


Figure 2. (a) Oscillations and (b) K and $|\Delta_s|$ of Wide Band Power Amplifier

RING OSCILLATOR

The second circuit example with the chosen element values is shown schematically in Figure 4. It is a very simple model of a ring oscillator which oscillates at ~1.43 GHz. A plot of K and $|\Delta_s|$ vs. frequency shown in Figure 5 indicates that k>1 and $|\Delta_s|$<1 at all frequencies for the element values shown in Figure 4. The circuit is nevertheless highly unstable as indicated by a direct solution of the determinant of the 4X4 Y matrix which fully describes this circuit. The solution reveals the existence of two complex conjugate poles at the RHP locations of ~ (4.94 ± j8.97)10^9. The three other simple poles, -266.3x10^9, -43.2x10^9 and -2.58x10^9 are located in the LHP.

Another direct indication of the instability of the circuit can be obtained by a polar plot of a suitably chosen function of the system determinant. It is well known from complex function theory that for any given F (x+jy) in the complex plane, and a closed contour in that plane which encircles N_p poles and N_z zeroes, that $N = N_p - N_z$ where N is the number of times the polar plot of F(x+jy) encircles the origin. For our purpose, we choose the particular closed contour in the σ,ω plane which is

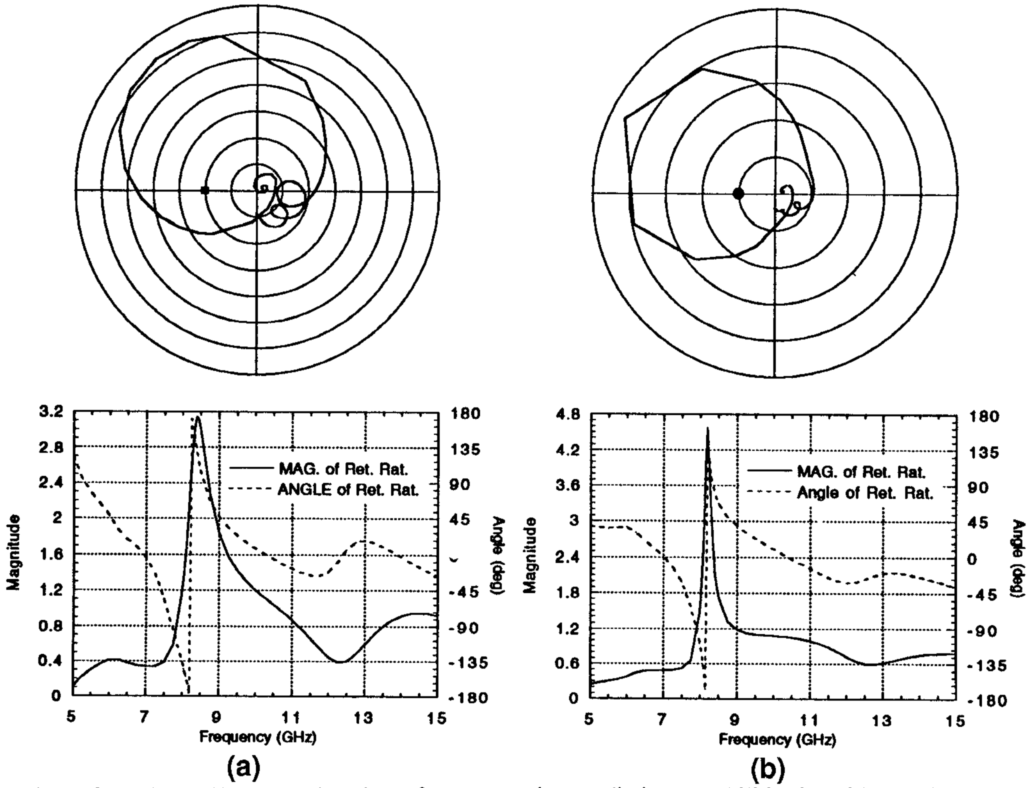


Figure 3. Polar and Rectangular Plots of Return Ratios Predicting Instabilities in Wide Band Power Amplifier Biased at Vds=4V, Vgs=-0.2V. (a) 800μm FETs (b) 1.2mm FETs

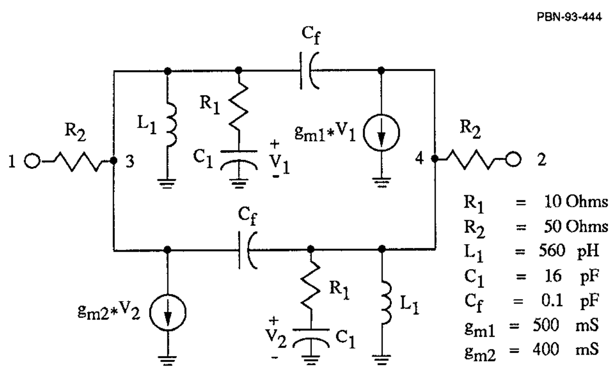


Figure 4. Schematic Diagram and Element Values of Ring Oscillator

comprised of the entire imaginary axis and the returning semicircle of infinite radius joining +jω and =jω. Since the response of any physically realizable circuit decays at ω~∞ and σ ~∞, no contribution arises from the returning semicircle, and therefore, the number of poles minus the number of zeroes $N_p - N_z$ which lie in the RHP can be determined from calculations along the ω axis alone. The values of the determinantal function at ω=±∞ will be identical and its contour will be a closed one.

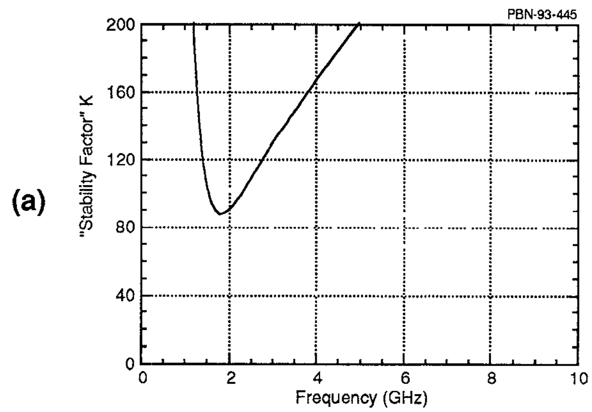
A properly chosen function of the determinant for plotting is a function which introduces no poles or zeroes in the RHP and which enables easy determination of the number of origin encirclements. Such a function is Δ / Δ_0 where Δ_0 is the determinant Δ with the active elements eliminated. A polar plot of the normalized 4x4 Y-matrix determinant is shown in Figure 6a. The two encirclements of the origin correspond to the two roots in the RHP calculated numerically.

The same method can be generalized to circuits which include any number of active elements. Either the determinant of the full n-node network or a determinant of a properly reduced m-nodes (where m<n) can be chosen for contouring or numerical solution.

The original network can be reduced by eliminating subnetworks provided they contain only passive circuit elements. The elimination procedure is based on the mathematical identity

$$\det \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \det |A-BD^{-1}C| \times \det|D|$$

where A, B, C, and D are submatrices of the network. The full network is represented by A, B, C, and D, the eliminated subnetwork is D, and the remaining subnetwork to be examined is $A-BD^{-1}C$.



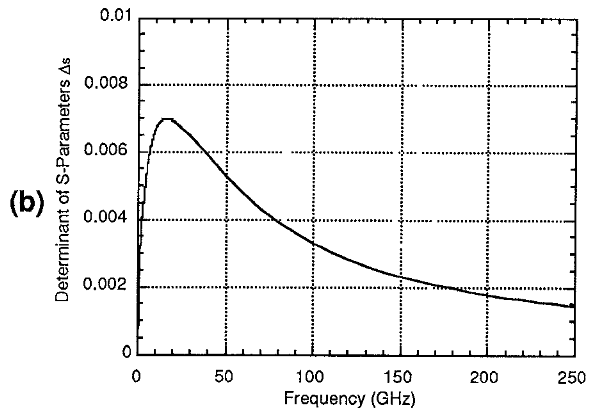


Figure 5. K and $|\Delta_s|$ of Ring Oscillator between Nodes 1 and 2

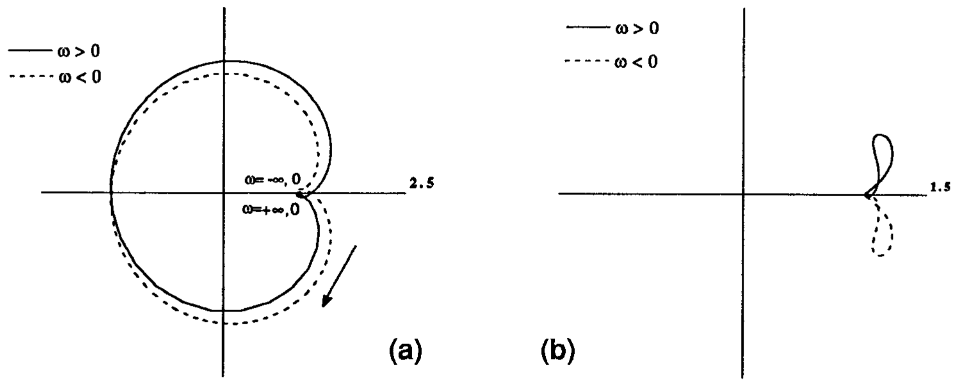


Figure 6. Contour Plots of Normalized Y Determinants of Ring Oscillator. (a) 4-Port Determinant. (b) 2-Port Determinant between Nodes 1 and 2.

In the case of networks which contain three terminal devices such as FETs, HBTs etc. that do not share common nodes, the minimum sized subnetworks which have to be examined for RHP zeroes, are of dimensions 3xn where n is the number of active devices. Any further reduction may introduce unresolvable unknowns as shown in the contour plot of Figure 6b. The figure shows a contour plot of the determinant of the Two-Port between nodes 1 and 2 which clearly does not encircle the origin. The reason for the apparent discrepancy is that the reduction prodedure creates a Two-Port determinant with unknown number of poles and zeroes in the RHP. This determinant can be shown to be equal to $\Delta_{4X4} / \Delta_{34}$ where Δ_{4X4} is the original 4-Port determinant and Δ_{34} is the determinant of the eliminated Two-Port between nodes 3 and 4. Since Δ_{34} contains active elements, it may in general, and actually does so in our case, contain zeroes in the RHP.

The method of contour plotting of functions of ω is very valuable even in todays environment where computers are available for solving circuit determinants. In large networks, especially ones which contain distributed elements, it can provide a quick answer to the question of the network's stability without resort to root solving methods in the complex s plane which are not readily available and are much more difficult to use.

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